Passage III

In 3 studies, students investigated the stiffness of rectangular metal beams (see Figure 1).

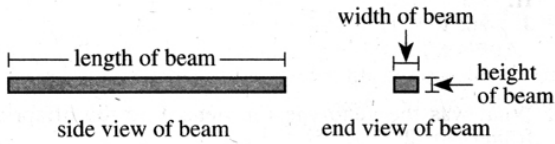side view of beam   end view of beam

Figure 1

Using the apparatus shown in Figure 2, the students deformed each beam under a variety of conditions.
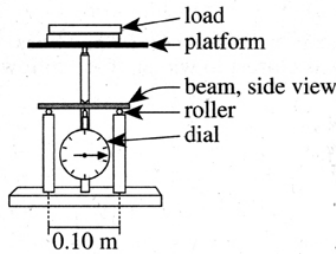

Figure 2

In each trial, the beam being tested was supported at 2 points that were 0.10 m apart and equidistant from the midpoint of the beam. The beam was subjected to a *load*, W, measured in newtons (N), at the midpoint of the beam. W was provided by weights placed on the platform of the apparatus. During deformation, each beam became slightly U-shaped (see Figure 3).
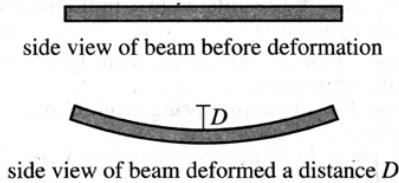

side view of beam before deformation

side view of beam deformed a distance D

Figure 3

A dial on the apparatus registered the distance, D, the beam was deformed in multiples of 10^{-6} m. After D was measured, the load was removed, and the beam returned to its original shape.

The intrinsic stiffness of the metal composing a beam was represented by *Young's modulus*, E. The effect of the width and height of a beam (see Figure 1) on D was represented by the quantity I, given in m^4.

Study 1

In Trials 1–4, students determined D for beams with different I (see Table 1). In every trial, $W = 20$ N and $E = 50 \times 10^9$ N/m^2.

	Table 1	
Trial	I $(10^{-9}$ m$^4)$	D $(10^{-6}$ m$)$
1	2.0	4.2
2	4.0	2.1
3	6.0	1.4
4	8.0	1.0

Study 2

In Trials 5–8, students determined D for beams composed of Metals S–V, respectively. Each metal had a different value of E (see Table 2). In every trial, $W = 20$ N and $I = 2.0 \times 10^{-9}$ m^4.

	Table 2		
Trial	Metal	E $(10^9$ N/m$^2)$	D $(10^{-6}$ m$)$
5	S	25	8.3
6	T	50	4.2
7	U	75	2.8
8	V	100	1.4

Study 3

In Trials 9–12, students determined D for different W (see Table 3). In every trial, $I = 2.0 \times 10^{-9}$ m^4 and $E = 50 \times 10^9$ N/m^2.

	Table 3	
Trial	W (N)	D $(10^{-6}$ m$)$
9	10	2.1
10	20	4.2
11	30	6.3
12	40	8.4

12. Which of the following diagrams of the side view of a beam correctly portrays the direction(s) of the 3 forces that the apparatus exerted on the beam?
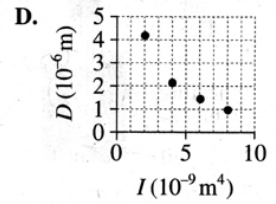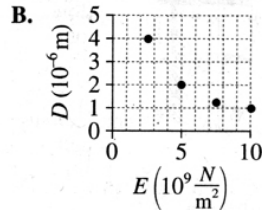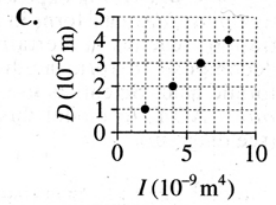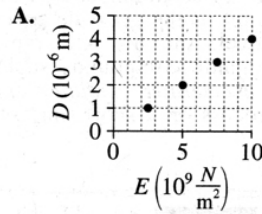
F.
up ↑

G.
up ↑

H.
up ↑

J.
up ↑

13. If, in Study 3, a trial had been conducted in which $W = 25$ N, D would most likely have been closest to which of the following?

A. 3.1×10^{-6} m
B. 5.2×10^{-6} m
C. 7.3×10^{-6} m
D. 9.4×10^{-6} m

14. If the amount of work done to deform a beam equaled $W \times D$, in which of the following trials was the amount of work the greatest?

F. Trial 2
G. Trial 4
H. Trial 6
J. Trial 8

15. The results of Study 1 are best represented by which of the following graphs?

A. *D* (10^{-6} m) vs. $E \left(10^9 \frac{N}{m^2}\right)$

B. *D* (10^{-6} m) vs. $E \left(10^9 \frac{N}{m^2}\right)$

C. *D* (10^{-6} m) vs. $I\,(10^{-9}\,m^4)$

D. *D* (10^{-6} m) vs. $I\,(10^{-9}\,m^4)$

16. The beam tested in Study 3 was most likely composed of which of the metals tested in Study 2 ?

F. Metal S
G. Metal T
H. Metal U
J. Metal V

17. Based on the results of Studies 1 and 2, for a given W, which of the following combinations of I and E would yield the stiffest beam?

	I ($10^{-9}\,m^4$)	E ($10^9\,N/m^2$)
A.	3.0	30
B.	3.0	40
C.	4.0	30
D.	4.0	40